

Gradients Areas Under Graphs

GCSE Maths · Written to the shared content for AQA, Edexcel and OCR

Name _____ Class _____ Date _____

1. Worked example

Estimate the area under $y = x^2$ from $x = 0$ to $x = 4$ using 4 strips of equal width.

Step 1 · Width $h = 1$; heights at $x = 0, 1, 2, 3, 4$

0, 1, 4, 9, 16

Step 2 · Trapezia: $\frac{1}{2} \times 1 \times (0 + 1) + \frac{1}{2}(1 + 4) + \frac{1}{2}(4 + 9) + \frac{1}{2}(9 + 16)$

0.5 + 2.5 + 6.5 + 12.5

Step 3 · Add

22

Answer: 22 square units (an overestimate)

Check: The exact area is $64/3 \approx 21.3$, a little less, because the curve bends below the trapezium tops ✓

2. Now try one with the steps

A cyclist speeds up from 0 to 10 m/s in 4 s, rides at 10 m/s for 6 s, then slows to rest in 5 s. Find the total distance and the first acceleration.

Step 1 · Area of the first triangle

Step 2 · Rectangle, then last triangle

Step 3 · Total, and gradient of the first part

Answer:

3. Practice

Show your working. The marks tell you how much is expected.

1. On a velocity–time graph, a car's speed rises steadily from 0 to 12 m/s in 3 s. Find its acceleration, in m/s^2 . [1]

2. How far does the car in question 1 travel in those 3 seconds, in metres? [2]

3. A velocity–time graph is flat at 8 m/s for 5 s. How far is travelled, in metres? [1]

4. What does the gradient of a distance–time graph represent? [1]
- _____
- _____
5. What does the area under a velocity–time graph represent? [1]
- _____
- _____
6. A strip under a curve runs from $x = 1$ to $x = 3$. The curve's heights there are 4 and 10. Find the area of the trapezium. [2]
- _____
- _____
7. Estimate the area under $y = x^2$ from $x = 0$ to $x = 3$ using 3 strips of equal width. [3]
- _____
- _____
- _____
8. Is your answer to question 7 an overestimate or an underestimate? Explain. [1]
- _____
- _____
9. Estimate the area under $y = 10 - x^2$ from $x = 0$ to $x = 3$ using 3 strips of equal width. [3]
- _____
- _____
- _____
10. Is your answer to question 9 an overestimate or an underestimate? [1]
- _____
- _____
11. A tangent to a curve at P passes through (0, 2) and (4, 10). Estimate the gradient of the curve at P. [1]
- _____
- _____
12. A particle travels $s = t^2 + 3t$ metres in t seconds. Find its average speed between $t = 2$ and $t = 5$, in m/s. [2]
- _____
- _____
13. A car accelerates steadily from rest to 20 m/s in 8 s, then slows steadily to rest in 12 s. Find the total distance travelled, in metres. [2]
- _____
- _____

14. Find the deceleration of the car in question 13, to 2 decimal places, in m/s^2 . [1]

15. A graph shows the depth of water in a bath against time. Part of the graph has a negative gradient. [2]
What is happening, and what does the gradient's value tell you?

4. Before you hand it in

Finding the 'gradient at $x = 2$ ' from the chord between $x = 1$ and $x = 3$ on a curve

A chord gives the AVERAGE rate. For the rate at one point, draw the tangent touching the curve there and use two points on the tangent.

Trapezium strip area = $h \times (a + b)$

Area of a trapezium = $\frac{1}{2} \times h \times (a + b)$. With 4 strips the half applies to every strip.

Reading the area under a distance–time graph

On a distance–time graph distance is read from the vertical axis and the gradient is speed. Area gives distance on a VELOCITY–time graph.

Using heights at $x = 0, 1, 2, 3$ for 4 strips from 0 to 4

n strips need $n + 1$ heights, from the first edge to the last: $x = 0, 1, 2, 3, 4$.