

Functions Iteration

GCSE Maths · Written to the shared content for AQA, Edexcel and OCR

Name _____ Class _____ Date _____

1. Worked example

Show that $x^3 + x - 5 = 0$ has a root between $x = 1$ and $x = 2$.

Step 1 · $f(1)$

$$1 + 1 - 5 = -3 \text{ (negative)}$$

Step 2 · $f(2)$

$$8 + 2 - 5 = 5 \text{ (positive)}$$

Step 3 · Conclusion

Sign change and f is continuous, so there is a root between 1 and 2

Answer: $f(1) = -3 < 0$ and $f(2) = 5 > 0$, so a root lies between 1 and 2

Check: The graph must cross $y = 0$ to get from -3 to 5 ✓

2. Now try one with the steps

Show that $x^3 + x - 5 = 0$ can be written $= \sqrt[3]{5 - x}$.

Step 1 · Subtract x and add 5

Step 2 · Cube root both sides

Answer:

3. Practice

Show your working. The marks tell you how much is expected.

1. $f(x) = x^3 - 2x - 5$. Find $f(2)$. [1]

2. For the same f , find $f(3)$. [1]

3. Explain why $x^3 - 2x - 5 = 0$ has a root between 2 and 3. [2]

4. Show that $x^2 - 5x + 2 = 0$ has a root between 4 and 5. [2]

5. Show that $x^2 - 5x + 2 = 0$ can be rearranged to $x = 5 - 2/x$. [2]

6. Using $x_{n+1} = 5 - 2/x_n$ with $x_0 = 4$, find x_1 . [1]

7. Continue: find x_2 to 4 decimal places. [1]

8. Continue: find x_3 to 4 decimal places. [1]

9. The exact root is $(5 + \sqrt{17})/2$. Give it to 3 decimal places and compare with your iterations. [1]

10. Show that $x^3 + 2x - 7 = 0$ has a root between 1 and 2. [2]

11. Using $x_{n+1} = \sqrt[3]{7 - 2x_n}$ with $x_0 = 1.5$, find x_1 to 4 decimal places. [1]

12. Continue: find x_2 to 4 decimal places. [1]

13. $f(x) = 3x + 2$. Find $f(4)$. [1]

14. Show that $x = \sqrt{2x + 3}$ is a rearrangement of $x^2 - 2x - 3 = 0$. [2]

15. Iterating $x_{n+1} = \sqrt{2x_n + 3}$ from $x_0 = 2$ gives values getting closer to a root of $x^2 - 2x - 3 = 0$. Which root? [2]

4. Before you hand it in

Rounding x_1 to 1.52 and using 1.52 to find x_2

Use the full calculator value (the ANS key). Round only the values you write down.

' $f(1) = -3$ and $f(2) = 5$, so the root is 1.5'

A sign change shows a root lies BETWEEN the values, not where. Iterate or narrow the interval to find it.

Showing a root is 1.52 (2 d.p.) by substituting 1.51 and 1.53

Use the bounds 1.515 and 1.525: a sign change between them proves the root rounds to 1.52.

Rearranging $x^3 + x - 5 = 0$ to $x = \sqrt[3]{5 - x}$

$x^3 = 5 - x$, so $x = \sqrt[3]{5 - x}$: the cube root applies to the whole of $5 - x$.