

Congruence Proofs

GCSE Maths · Written to the shared content for AQA, Edexcel and OCR

Name _____ Class _____ Date _____

1. Worked example

ABCD is a parallelogram. Prove that triangles ABC and CDA are congruent.

Step 1 · $AB = CD$

opposite sides of a parallelogram are equal

Step 2 · $BC = DA$

opposite sides of a parallelogram are equal

Step 3 · $AC = CA$

common side

Answer: Triangles ABC and CDA are congruent (SSS)

Check: Each line has a reason, and the condition is named ✓

2. Now try one with the steps

Triangle ABC has $AB = AC$. M is the midpoint of BC. Prove that angle ABC = angle ACB.

Step 1 · $AB = AC$

Step 2 · $BM = CM$

Step 3 · $AM = AM$

Step 4 · So corresponding angles are equal

Answer:

3. Practice

Show your working. The marks tell you how much is expected.

1. Name the four conditions for two triangles to be congruent. [2]

2. Explain why three equal angles do not prove that two triangles are congruent. [1]

3. One triangle has sides 5 cm, 7 cm, 9 cm and another has sides 9 cm, 5 cm, 7 cm. Are they congruent? Give the condition. [1]
4. Two triangles each have sides 6 cm and 8 cm with an included angle of 40° . Which condition shows they are congruent? [1]
5. Triangle PQR is congruent to triangle XYZ, with $P \leftrightarrow X$, $Q \leftrightarrow Y$, $R \leftrightarrow Z$. $PQ = 5$ cm and angle $Q = 50^\circ$. Write down XY and angle Y. [2]
6. Explain why two sides and a non-included angle (SSA) is not enough to prove congruence. [2]
7. ABCD is a kite with $AB = AD$ and $CB = CD$. Prove that triangles ABC and ADC are congruent. [3]
8. O is the centre of a circle and AB is a chord with midpoint M. Prove that OM is perpendicular to AB. [4]
9. ABCD is a square. E is the midpoint of AB and F is the midpoint of BC. Prove that triangles DAE and ABF are congruent. [3]
10. Two right-angled triangles each have a hypotenuse of 13 cm and another side of 5 cm. Are they congruent? Find the third side. [2]

11. In triangles ABC and DEF, angle A = angle D = 50° , angle B = angle E = 60° and AB = DE = 8 cm. [2]
Are they congruent? Find angle C.

12. In triangle ABC, AB = AC. D is on BC and AD bisects angle BAC. Prove that BD = CD. [3]

13. In an 'hourglass' of similar triangles ABC and EDC, AB = 6 cm, ED = 9 cm and AC = 4 cm. Find EC, [2]
in cm.

14. In triangle ABC, D is on AB and E is on AC with DE parallel to BC. Prove that triangles ADE and [3]
ABC are similar.

15. Explain how congruent triangles prove that opposite angles of a parallelogram are equal. [2]

4. Before you hand it in

'They are congruent because they look the same'

List three equal parts, each with a reason (given, common, alternate angles ...), then name the condition.

Using AAA to claim congruence

Equal angles show the triangles are similar. Congruence needs at least one pair of equal corresponding sides.

Using SAS when the angle is not between the two sides

SSA does not prove congruence. Check the angle is formed by the two sides you are using.

Matching the wrong vertices (writing $ABC \cong DFE$ when B matches E)

Write the triangles so matching letters are in matching positions; then corresponding sides and angles can be read off correctly.