

Conditional Probability

GCSE Maths · Written to the shared content for AQA, Edexcel and OCR

Name _____ Class _____ Date _____

1. Worked example

Using the starting table (bus & late 12, bus & on time 48, walk & late 6, walk & on time 54), find $P(\text{late} \mid \text{bus})$.

Step 1 · Restrict to bus users

$$12 + 48 = 60$$

Step 2 · Late among them

$$12$$

Step 3 · $P(\text{late} \mid \text{bus})$

$$12/60 = 1/5$$

Answer: 1/5

Check: $P(\text{late})$ overall = $18/120 = 3/20$, which is different, so not independent ✓

2. Now try one with the steps

In a Venn diagram of 50 people: 8 in A only, 12 in $A \cap B$, 10 in B only, 20 in neither. Find $P(A \mid B)$ and $P(B \mid A)$.

Step 1 · $n(B)$

Step 2 · $P(A \mid B)$

Step 3 · $n(A) = 20$

Answer:

3. Practice

Show your working. The marks tell you how much is expected.

1. In a class of 30, 18 have a pet. Of those with a pet, 6 have a cat. A pupil with a pet is chosen at random. Find $P(\text{cat} \mid \text{pet})$. [1]

2. Two-way table: Year 10 — 25 like football, 15 don't; Year 11 — 20 like football, 20 don't. Find $P(\text{likes football} \mid \text{Year 10})$. [2]

3. Same table: find $P(\text{Year 10} \mid \text{likes football})$. [2]

4. Same table: find $P(\text{Year 11} \mid \text{doesn't like football})$. [2]

5. Venn diagram of 40: A only 10, $A \cap B$ 6, B only 14, neither 10. Find $P(A \mid B)$. [2]

6. Same Venn diagram: find $P(B \mid A)$. [2]

7. Same Venn diagram: find $P(A \mid B')$ (A given NOT B). [3]

8. A box has 6 milk and 4 dark chocolates. Two are eaten without replacement. Find $P(\text{second is dark} \mid \text{first is milk})$. [2]

9. Same box: find $P(\text{both dark})$. [2]

10. $P(A) = 0.5$ and $P(B \mid A) = 0.6$. Find $P(A \text{ and } B)$. [1]

11. $P(A \cap B) = 0.12$ and $P(B) = 0.4$. Find $P(A \mid B)$. [2]

12. $P(A) = 0.3$, $P(B) = 0.5$ and $P(A \cap B) = 0.15$. Are A and B independent? Explain. [2]

13. $P(\text{snow}) = 0.2$. If it snows, $P(\text{school closes}) = 0.5$; otherwise 0.05. Find $P(\text{school closes})$. [3]

14. Using the previous question, find $P(\text{snow} \mid \text{school closes})$. Give your answer fraction. [3]

15. A bag has 3 green and 2 yellow sweets. Two are taken without replacement. Given that at least one is green, find the probability both are green. [4]

4. Before you hand it in

$P(\text{late} \mid \text{bus}) = 12/120$

Given bus means only bus users count: divide by the bus total, 60.

$P(A \mid B) = P(B \mid A)$

The condition after the bar sets the denominator. $P(A \mid B)$ uses $n(B)$; $P(B \mid A)$ uses $n(A)$.

Keeping the same probabilities on the second branches without replacement

Reduce the numerator and denominator after each pick.

Assuming events are independent

Only multiply $P(A) \times P(B)$ if independence is stated or shown. Otherwise use $P(A) \times P(B \mid A)$.