

Completing the Square

GCSE Maths · Written to the shared content for AQA, Edexcel and OCR

Name _____ Class _____ Date _____

1. Worked example

Write $x^2 + 8x + 5$ in the form $(x + p)^2 + q$.

Step 1 · Halve the x coefficient

$$8 \div 2 = 4 \rightarrow (x + 4)^2$$

Step 2 · $(x + 4)^2$ gives an extra $4^2 = 16$, so subtract it

$$(x + 4)^2 - 16 + 5$$

Step 3 · Simplify

$$(x + 4)^2 - 11$$

Answer: $(x + 4)^2 - 11$

Check: $(x + 4)^2 - 11 = x^2 + 8x + 16 - 11 = x^2 + 8x + 5$ ✓

2. Now try one with the steps

Solve $x^2 - 6x + 4 = 0$ by completing the square. Give exact answers.

Step 1 · Complete the square

Step 2 · Set equal to 0 and rearrange

Step 3 · Square root — both signs

Answer:

3. Practice

Show your working. The marks tell you how much is expected.

1. Write $x^2 + 6x$ in the form $(x + p)^2 + q$. [2]

2. Write $x^2 + 4x + 7$ in the form $(x + p)^2 + q$. [2]

3. Write $x^2 - 10x + 30$ in the form $(x + p)^2 + q$. [2]

4. Write $x^2 - 2x - 8$ in the form $(x + p)^2 + q$. [2]

5. Write $x^2 + 5x + 1$ in the form $(x + p)^2 + q$. [3]

6. $x^2 - 8x + 3 = (x - p)^2 - q$. Find q . [2]

7. Find the coordinates of the turning point of $y = x^2 - 4x + 1$. [2]

8. Find the minimum value of $x^2 + 12x + 40$. [2]

9. Solve $x^2 + 4x - 1 = 0$ by completing the square. Give exact answers. [3]

10. Solve $x^2 - 10x + 18 = 0$ by completing the square. Give exact answers. [3]

11. Write $2x^2 + 8x + 3$ in the form $a(x + p)^2 + q$. [3]

12. Write $3x^2 - 6x + 10$ in the form $a(x + p)^2 + q$. [3]

13. Show that $x^2 - 6x + 11 > 0$ for every value of x . [3]

14. Find the turning point of $y = -x^2 + 6x - 4$ and say whether it is a maximum or a minimum. [3]

15. $x^2 + bx + c \equiv (x - 4)^2 + 3$. Find b and c . [2]

4. Before you hand it in

$$x^2 + 8x + 5 = (x + 8)^2 - 64 + 5$$

$(x + 8)^2$ gives $16x$, not $8x$. Halve: $(x + 4)^2 - 16 + 5 = (x + 4)^2 - 11$.

$$x^2 + 8x + 5 = (x + 4)^2 + 5$$

$(x + 4)^2$ already contains $+16$, which was not in the original. Subtract it: $(x + 4)^2 - 11$.

Turning point of $(x + 4)^2 - 11$ is $(-4, -11)$

The bracket is zero when $x = -4$, so the turning point is $(-4, -11)$.

$$2(x^2 - 6x) + 7 = 2(x - 3)^2 - 9 + 7$$

Keep the square brackets: $2[(x - 3)^2 - 9] + 7 = 2(x - 3)^2 - 18 + 7 = 2(x - 3)^2 - 11$.