

Algebraic Proof

GCSE Maths · Written to the shared content for AQA, Edexcel and OCR

Name _____ Class _____ Date _____

1. Worked example

Prove that the sum of any three consecutive integers is a multiple of 3.

Step 1 · Let the integers be

$$n, n + 1, n + 2$$

Step 2 · Add

$$3n + 3$$

Step 3 · Factorise

$$3(n + 1)$$

Answer: $3(n + 1)$ has a factor of 3, so the sum is always a multiple of 3

Check: e.g. $7 + 8 + 9 = 24 = 3 \times 8$ ✓ (an illustration, not the proof)

2. Now try one with the steps

Prove that $(2n + 1)^2 - (2n - 1)^2$ is a multiple of 8 for all integers n .

Step 1 · Expand each square

Step 2 · Simplify carefully

Step 3 · Conclude

Answer:

3. Practice

Show your working. The marks tell you how much is expected.

1. Write an expression for any even number, using the integer n . [1]

2. Write an expression for any odd number, using the integer n . [1]

3. Write expressions for three consecutive integers. [1]

4. Give a counterexample to 'all prime numbers are odd'. [1]

5. Give a counterexample to ' n^2 is always greater than n '. [1]

6. Prove that the sum of two consecutive integers is always odd. [2]

7. Prove that the sum of any two odd numbers is even. [3]

8. Prove that the sum of four consecutive integers is always even. [3]

9. Prove that $(n + 3)^2 - (n + 1)^2 = 4(n + 2)$. [3]

10. Prove that the difference between the squares of two consecutive integers is always odd. [3]

11. Prove that $(2n + 3)^2 - (2n - 3)^2$ is a multiple of 12. [3]

12. Prove that $x^2 - 8x + 20 > 0$ for all values of x . [3]

13. Prove that the product of two consecutive even numbers is a multiple of 8. [4]

14. Prove that the square of any odd number is one more than a multiple of 8. [4]

15. Is the sum of any three consecutive odd numbers always a multiple of 3? Prove or disprove. [3]

4. Before you hand it in

'3 + 5 = 8, 7 + 9 = 16, 11 + 1 = 12, so the sum of two odd numbers is always even'

Examples only show it works in those cases. Use general expressions: $(2a + 1) + (2b + 1) = 2(a + b + 1)$.

Two odd numbers written as $2n + 1$ and $2n + 1$

That makes both numbers the same. Use $2a + 1$ and $2b + 1$ (or $2n + 1$ and $2m + 1$).

$$(2n + 1)^2 - (2n - 1)^2 = 4n^2 + 4n + 1 - 4n^2 - 4n + 1 = 2$$

Put the expansion in a bracket: $-(4n^2 - 4n + 1) = -4n^2 + 4n - 1$. The answer is $8n$.

Stopping at $6n + 3$ without a conclusion

Factorise and say it: $6n + 3 = 3(2n + 1)$, which is a multiple of 3.